Non-threshold D-brane bound states and black holes with non-zero entropy

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Abstract

We start with BPS-saturated configurations of two (orthogonally) intersecting M-branes and use the electro-magnetic duality or dimensional reduction along a boost, in order to obtain new p-brane bound states. In the first case the resulting configurations are interpreted as BPS-saturated non-threshold bound states of intersecting p-branes, and in the second case as p-branes intersecting at angles and their duals. As a by-product we deduce the enhancement of supersymmetry as the angle approaches zero. We also comment on the D-brane theory describing these new bound states, and a connection between the angle and the world-volume gauge fields of the D-brane system. We use these configurations to find new embeddings of the four and five dimensional black holes with non-zero entropy, whose entropy now also depends on the angle and world-volume gauge fields. The corresponding D-brane configuration sheds light on the microscopic entropy of such black holes.

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I. INTRODUCTION

The existence of BPS-saturated configurations of superstring theory has become a corner stone in establishing and testing the different non-perturbative superstring dualities ([1,2] and references there in). The importance of such BPS-saturated states relies on the fact that (for $N \geq 4$ supersymmetry) their classical properties do not receive quantum corrections and thus one can follow their properties to the strong coupling regime.

As a consequence, over the last year there has been a dramatic progress in the understanding of different BPS-saturated p-brane solutions in supergravity theories. In particular, p-brane configurations within $D = 11$ supergravity are of special interest, since the latter theory is the effective field theory limit of the conjectured M-theory [2,3]. By intersecting different p-brane configurations, each of them preserving $1/2$ of the maximal supersymmetry [4–6], it has been possible to construct new solutions preserving a lower fraction of the maximal supersymmetry [7–12]. These configurations can be reduced to lower dimensions to obtain supersymmetric p-brane solutions of superstring theories [13–16]. The BPS-saturated (basic) p-brane configurations that intersect orthogonally, correspond to the marginally bound configurations.

On the other hand, there exist non-marginally (non-threshold) bound BPS-saturated configurations among the composite p-branes. Typically these configurations interpolate between different type of configurations. To be definite, consider the bound state of a D-2-brane lying within a D-4-brane as an example. This configuration can be obtained by reducing the M-theory bound state interpreted as a membrane lying within a five-brane [17], which in turn arises as a consequence of the $SL(2,\mathbb{Z})$ electromagnetic duality of the $D = 8$ Type II superstring theory [1,6]. The same configuration arises by performing T-duality at an angle on the Type IIB 3-brane solution [18,19]. Other bound states can be obtained by reducing a M-theory configuration at an angle or along a boost [20,21], or by performing a T-duality operation along a boost [19].

The aim of this paper is to find new non-threshold BPS-saturated states of M-theory and superstring theories by using these (solution generating) techniques. In particular, we will be interested in finding new D-brane [22] configurations that may be relevant for the black hole entropy counting [23–27]. Therefore most of the solutions presented will be bound states of D-branes. Their uplift to eleven dimensions and their M-theory interpretation should be straightforward. We will also comment on the D-brane theory associated with the new bound states.

The paper is organised as follows. In Section II we shall study the action of the $D = 8$ type II superstring theory U-duality subgroup $SL(2,\mathbb{Z})$ on the intersecting M-branes. The resulting configurations will be reduced to D-brane bound states of the Type IIA theory. In Section III, by reducing along a boost intersecting configurations of two M-branes, we will be able to deduce the supergravity solutions that are interpreted as two D-p-branes intersecting at an angle [28,29], as well as other p-branes intersecting at an angle. In both Sections II and III we will find new embeddings of the supersymmetric $D = 4$ [30–33] and $D = 5$ [34] black holes with non-zero entropy. In section IV we shall give some concluding remarks.

Before proceeding, we first settle our notation and conventions. We will be representing the orthogonal intersection of two configurations A and B by $A \perp B$, and a (non-threshold)

configuration interpolating between A and B by $(A|B)$. For example, in this notation $(2|5)$ represents a membrane lying within a five-brane and $(2|5) \perp (2|5)$ the orthogonal intersection of two of these bound states. We will work with the following action for the bosonic sector of $D = 11$ supergravity

$$I_{11} = \int d^{11}x \sqrt{-g} \left[R - \frac{1}{2.4!} \mathcal{F}^2 \right] + \frac{1}{6} \int \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{A} \, , \tag{I.1}$$

where $\mathcal{F} = d\mathcal{A}$ and $\mathcal{A}$ is a 3-form field. The p-brane supergravity background fields are usually expressed in terms of harmonic functions on the d-dimensional overall transverse space of the configuration $\mathbb{E}^d$. For simplicity, we will consider the case where the harmonic functions are centered at the origin, i.e.

$$H_i = 1 + \frac{\alpha_i}{|\mathbf{x}|^{d-2}} \, , \tag{I.2}$$

where $\mathbf{x}$ are the coordinates on $\mathbb{E}^d$. All the solutions will be presented in terms of the corresponding harmonic functions, therefore the multi-centered case follows immediately.

The electric charges are defined by

$$Q_i = \frac{1}{(d-2)A_{d-1}V_i} \int_{\Sigma_i} \star \mathcal{F} \, , \tag{I.3}$$

where A_{d-1} is the unit $(d-1)$-sphere volume, V_i is the volume of the compact space along which the corresponding membrane is delocalized and Σ_i is the product of this compact space with the asymptotic $(d-1)$-sphere. Similarly, the magnetic charges are defined by

$$P_i = \frac{1}{(d-2)A_{d-1}V_i} \int_{\Sigma_i} \mathcal{F} \, , \tag{I.4}$$

where V_i is the volume of the compact space along which the corresponding five-brane is delocalized and Σ_i is the product of this compact space with the asymptotic $(d-1)$-sphere. Similar definitions hold for the electric and magnetic charges associated with a given n-form field strength of the lower dimensional theories.

II. ELECTROMAGNETIC DUALITY AND BOUND STATES OF INTERSECTING BRANES

Reduction of the M-theory on a 3-torus yields the $D = 8$ Type II superstring theory. The corresponding U-duality group is $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$ [1]. In this section we will study the action of the $SL(2, \mathbb{Z})$ electro-magnetic duality group on the intersecting M-brane configurations. Following [6] we will start by presenting the action of the $SL(2, \mathbb{R})$ electro-magnetic duality group on the bosonic sector of the $D = 8$, $N = 2$ supergravity. The latter theory admits a consistent truncation with the following bosonic field content: a metric g_{ab}, a scalar σ, a pseudo scalar ρ and a 3-form gauge potential $\mathcal{A}_3$. This truncation can be embedded in the $D = 11$ supergravity by performing the Kałuża-Klein Ansätze

$$ds_{11}^2 = e^{\frac{2}{3}\sigma}ds_8^2 + e^{-\frac{4}{3}\sigma}d\mathbf{u}.d\mathbf{u} \ ,$$

$$\mathcal{F} = \mathcal{F}_4 + d\rho \wedge du_1 \wedge du_2 \wedge du_3 \ ,$$

(II.1)

where u_i with $i = 1, 2, 3$ are the coordinates on the 3-torus. Next, define the following complex scalar field λ and 4-form field strength $\mathcal{G}$

$$\lambda = \rho + ie^{-2\sigma} \ ,$$
$$\mathcal{G} = e^{-2\sigma} \star \mathcal{F}_4 - \rho\mathcal{F}_4 \ ,$$

(II.2)

where $\star$ is the dual with respect to the eight-dimensional metric in (II.1). The electro-magnetic duality acts on these fields by the $SL(2, \mathbb{R})$ transformation

$$\lambda \rightarrow \frac{a\lambda+b}{c\lambda+d} \ ,$$
$$(\mathcal{F}, \mathcal{G}) \rightarrow (\mathcal{F}, \mathcal{G}) \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \ .$$

(II.3)

In the quantum theory the $SL(2, \mathbb{R})$ group is broken to $SL(2, \mathbb{Z})$, i.e. a, b, c, d are integers satisfying $ad - bc = 1$. In the following subsections we will consider the action of the $U(1)$ subgroup, whose matrix elements are

$$\begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \ ,$$

(II.4)

on the intersecting M-branes. The full $SL(2, \mathbb{Z})$ multiplets can be found by acting with an $SL(2, \mathbb{Z})$ transformation on the set of configurations obtained by acting with the $U(1)$ transformation (II.4) on the intersecting M-branes we are considering [6]. Since this $SL(2, \mathbb{Z})$ transformation changes only the asymptotic values of the fields, we will restrict ourselves to configurations which are obtained by $U(1)$ transformations only, i.e. we confine ourselves to the study of configurations with the canonical asymptotic values of the scalar fields.

A. The $(5 \perp 5|2 \perp 2)$ configuration

Let us start by considering the $5 \perp 5$ M-theory solution [7,8]

$$ds_{11}^2 = (H_1 H_2)^{\frac{2}{3}} \left[(H_1 H_2)^{-1} \left(-dt^2 + d\mathbf{u}.d\mathbf{u} \right) \right.$$

$$\left. +H_1^{-1}ds^2(\mathbb{E}_1^2) + H_2^{-1}ds^2(\mathbb{E}_2^2) + ds^2(\mathbb{E}^3) \right] \ ,$$

(II.5)

$$\mathcal{F} = -\star dH_1 \wedge \epsilon(\mathbb{E}_2^2) - \star dH_2 \wedge \epsilon(\mathbb{E}_1^2) \ ,$$

where $\epsilon(\mathbb{E}_i^2)$ is the volume form on the corresponding two-dimensional Euclidean space and $\star$ is the dual with respect to the Euclidean metric in $\mathbb{E}^3$. The functions H_i are harmonic functions on the overall transverse space $\mathbb{E}^3$ as described at the end of the introduction. Next, perform a $U(1)$ electro-magnetic duality transformation by using the formulae given in the previous subsection. The resulting configuration interpolates between the original

$5 \perp 5$ and the $2 \perp 2$ configurations. In our notation it is represented by $(5 \perp 5|2 \perp 2)$. The corresponding background fields are given by

$$ds_{11}^2 = (\tilde{H}_{12}H_1H_2)^{\frac{1}{3}} \left[-(H_1H_2)^{-1}dt^2 + \tilde{H}_{12}^{-1}ds^2(\mathbb{E}_1^3) \right.$$

$$\left. +H_1^{-1}ds^2(\mathbb{E}_1^2) + H_2^{-1}ds^2(\mathbb{E}_2^2) + ds^2(\mathbb{E}_2^3) \right] \,,$$

$$\mathcal{F} = -\cos\zeta \left[\star dH_1 \wedge \epsilon(\mathbb{E}_2^2) + \star dH_2 \wedge \epsilon(\mathbb{E}_1^2) \right] \tag{II.6}$$

$$-\sin\zeta \left[dH_1^{-1} \wedge \epsilon(\mathbb{M}_1^3) + dH_2^{-1} \wedge \epsilon(\mathbb{M}_2^3) \right]$$

$$+\cos\zeta \sin\zeta \, d\left(\frac{H_1H_2-1}{\tilde{H}_{12}}\right) \wedge \epsilon(\mathbb{E}_1^3) \,,$$

where $\tilde{H}_{12} = \sin^2\zeta + \cos^2\zeta \, H_1H_2$ and $\epsilon(\mathbb{M}_i^3)$ is the volume form on the Minkowski space $\mathbb{R} \times \mathbb{E}_i^2$. If $\sin\zeta = 0$ we obtain the $5 \perp 5$ solution and if $\cos\zeta = 0$ the $2 \perp 2$ solution.

The ADM mass density of this composite configuration can be easily calculated [35]. The result is

$$\frac{M}{A_2} = \sqrt{P_1^2 + Q_1^2} + \sqrt{P_2^2 + Q_2^2} \,, \tag{II.7}$$

where Q_i and P_i are, respectively, the electric and magnetic charges of the constituent M-branes and are given by

$$P_i = \cos\zeta \, \alpha_i \,, \quad Q_i = \sin\zeta \, \alpha_i \,, \quad i = (1,2). \tag{II.8}$$

Consider now the reduction on S^1 of the solution (II.6) along one direction of $\mathbb{E}_1^3$. It is straightforward to obtain the corresponding Type IIA theory background fields. This solution describes a new bound state of D-branes, and it is represented in our notation as $(4 \perp 4|2 \perp 2)$. It interpolates between the $4 \perp 4$ D-brane configuration (say, n_1 D-4-branes at the origin lying along the Y^1, Y^2, Y^3, Y^4 directions and n_2 D-4-branes at the origin lying along the Y^1, Y^2, Y^5, Y^6 directions) and the $2 \perp 2$ D-brane configuration (n_1 D-2-branes at the origin lying along the Y^3, Y^4 directions and n_2 D-2-branes at the origin lying along the Y^5, Y^6 directions). As mentioned in [19] such configurations can be obtained by performing a T-duality at an angle on orthogonally intersecting configurations.[1] In fact, consider the Type IIB $3 \perp 3$ D-brane configuration (n_1 D-3-branes at the origin lying along the Y^1, Y^3, Y^4 directions and n_2 D-3-branes at the origin lying along the Y^1, Y^5, Y^6 directions) and rotate the configuration in the Y^1, Y^2 plane. This D-brane configuration can be presented as parallel to the following directions:

$$\begin{aligned} 3_1 &: Y^3, Y^4, Y^2 = \tan\zeta \, Y^1 \,, \\ 3_2 &: Y^5, Y^6, Y^2 = \tan\zeta \, Y^1 \,. \end{aligned} \tag{II.9}$$

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[1]This result is not surprising since within superstring theory the $SL(2,\mathbb{Z})$ electro-magnetic duality group is a perturbative symmetry [1].

Taking the Y^1 and Y^2 directions to be circles of radius R_1 and R_2, the angle ζ obeys the quantisation condition $\tan \zeta = \frac{\omega^2}{\omega^1} \frac{R_2}{R_1}$. Thus, the D-3-branes are wrapped along the (ω^1, ω^2) cycle of the $Y^1 Y^2$ 2-torus. Both D-3-branes in this configuration can be seen as lying along the Y^1 direction but with the $U(1)$ center of the $U(n_i)$ worldvolume scalar field Y^2 turned on as in (II.9) [36].[2] Similar comments will apply to other configurations presented in this paper.

Applying T-duality along the Y^2 direction [38] of the configuration (II.9) we obtain the D-brane theory corresponding to the $(4 \perp 4|2 \perp 2)$ bound state

$$
\begin{aligned}
4_1 : \ & Y^1, Y^2, Y^3, Y^4 \ , \\
& F_{12} = \tan \zeta \ , \\
4_2 : \ & Y^1, Y^2, Y^5, Y^6 \ , \\
& F_{12} = \tan \zeta \ ,
\end{aligned}
\tag{II.10}
$$

where F is the world-volume 2-form gauge field strength and we have considered its $U(1)$ factor in the decomposition $U(n_i) = (U(1) \times SU(n_i))/Z_{n_i}$. When this field is turned on as in (II.10), it ensures the coupling of a D-$(p+2)$-brane to a D-p-brane [39]. Note that it is essential that both world-volume gauge fields are turned on as in (II.10). In fact, had we turned on just the world-volume gauge field associated with the first group of D-4-branes the corresponding configuration would have been interpreted as $(4|2) \perp 4$ in a manner incompatible with the D-brane intersecting rules [38]. The configuration (II.10) provides another example of how the world-volume gauge field is essential for ensuring supersymmetry of the configuration [40]. More generally, by applying T-duality along transverse directions to the configuration (II.9), we have the non-threshold D-brane bound states

$$
\Big((p+2) \perp (p+2) | p \perp p \Big) \ ,
\tag{II.11}
$$

for $2 \leq p \leq 5$.

B. The $(2 \perp 5|5 \perp 2)$ configuration

For the sake of completeness, we will now repeat the previous analysis for the $2 \perp 5$ configuration of M-theory [8]

$$
\begin{aligned}
ds_{11}^2 = \ & H_1^{\frac{1}{3}} H_2^{\frac{2}{3}} \Big[(H_1 H_2)^{-1} ds^2(\mathbb{M}^2) + H_1^{-1} dy_2^2 \\
& + H_2^{-1} \left(dy_3^2 + d\mathbf{u}.d\mathbf{u} \right) + ds^2(\mathbb{E}^4) \Big] \ , \\
\mathcal{F} = \ & -dH_1^{-1} \wedge \epsilon(\mathbb{M}^2) \wedge dy_2 - \star dH_2 \wedge dy_2 \ ,
\end{aligned}
\tag{II.12}
$$

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[2]In the cases where the winding number ω^1 is bigger than one it is necessary to introduce extra $U(\omega^1)$ indices for the world-volume fields [37].

where we use the same notation as in (II.5). Performing a $U(1)$ electro-magnetic duality transformation as in the previous subsection we obtain the following solution

$$ds_{11}^2 = \left(\tilde{H}_{12} H_1 H_2\right)^{\frac{1}{3}} \left[(H_1 H_2)^{-1} ds^2(\mathbb{M}^2) + H_1^{-1} dy_2^2\right.$$

$$\left. + H_2^{-1} dy_3^2 + \tilde{H}_{12}^{-1} ds^2(\mathbb{E}^3) + ds^2(\mathbb{E}^4)\right] \ ,$$

$$\mathcal{F} = -\cos\zeta \left[dH_1^{-1} \wedge \epsilon(\mathbb{M}^2) \wedge dy_2 + \star dH_2 \wedge dy_2\right]$$

$$+ \sin\zeta \left[\star dH_1 \wedge dy_3 + dH_2^{-1} \wedge \epsilon(\mathbb{M}^2) \wedge dy_3\right] \qquad \text{(II.13)}$$

$$+ \cos\zeta \sin\zeta \ d\left(\tfrac{H_2 - H_1}{\tilde{H}_{12}}\right) \wedge \epsilon(\mathbb{E}^3) \ ,$$

where $\tilde{H}_{12} = \sin^2\zeta \ H_1 + \cos^2\zeta \ H_2$. In both limiting cases, $\sin\zeta = 0$ and $\cos\zeta = 0$, this solution reduces to the $2 \perp 5$ solution. However, the constituent M-branes are located along different directions. Also, as ζ varies the electric and magnetic charges are interchanged. The ADM mass density of our solution is again given (up to a constant) by (II.7). However, the electromagnetic charges are now given by

$$Q_1 = \cos\zeta \ \alpha_1 \ , \quad P_1 = -\sin\zeta \ \alpha_1 \ ,$$

$$P_2 = \cos\zeta \ \alpha_2 \ , \quad Q_2 = \sin\zeta \ \alpha_2 \ . \qquad \text{(II.14)}$$

We proceed by describing the Type IIA D-brane bound state associated with the solution (II.13). Reducing along one direction of $\mathbb{E}^3$ we get the D-brane bound state $(2 \perp 4|4 \perp 2)$. It interpolates between the $2 \perp 4$ configuration (say, n_1 D-2-branes lying along the Y^1, Y^2 directions and n_2 D-4-branes lying along the Y^1, Y^3, Y^4, Y^5 directions) and the $4 \perp 2$ D-brane configuration (n_1 D-4-branes lying along the Y^1, Y^2, Y^4, Y^5 directions and n_2 D-2-branes lying along the Y^1, Y^3 directions). Again, we can derive this configuration by performing T-duality at an angle on the Type IIB $3 \perp 3$ D-brane configuration. Starting with n_1 D-3-branes lying along the Y^1, Y^2, Y^4 directions and n_2 D-3-branes lying along the Y^1, Y^3, Y^5 directions and rotating the configuration in the $Y^4 Y^5$ plane we have

$$\begin{aligned} 3_1 &: Y^1, Y^2, Y^4 = \cot\zeta \ Y^5 \ , \\ 3_2 &: Y^1, Y^3, Y^4 = -\tan\zeta \ Y^5 \ . \end{aligned} \qquad \text{(II.15)}$$

Applying T-duality along Y^4 we obtain the D-brane theory corresponding to the $(2 \perp 4|4 \perp 2)$ bound state

$$\begin{aligned} 4_1 &: Y^1, Y^2, Y^4, Y^5 \ , \\ & \quad F_{45} = -\cot\zeta \ , \\ 4_2 &: Y^1, Y^3, Y^4, Y^5 \ , \\ & \quad F_{45} = \tan\zeta \ . \end{aligned} \qquad \text{(II.16)}$$

Once again, the turned on world-volume gauge fields in (II.16) ensure that the configuration remains supersymmetric. More generally, there are the non-threshold bound states of D-branes

$$\left(p \perp (p+2)|(p+2) \perp p\right) , \tag{II.17}$$

for $1 \leq p \leq 6$.

C. Black holes with non-zero entropy

We proceed by presenting new embeddings of the supersymmetric black holes with non-zero entropy. Consider the solution described by (II.13). There is a direction common to all the constituent M-branes. Therefore we can add a plane wave along this direction [8]. The resulting solution is described by the metric

$$ds_{11}^2 = \left(\tilde{H}_{12}H_1H_2\right)^{\frac{1}{3}}[(H_1H_2)^{-1}\left(-dt^2 + dy_1^2 + (H_3 - 1)(dy_1 \pm dt)^2\right)$$
$$+H_1^{-1}dy_2^2 + H_2^{-1}dy_3^2 + \tilde{H}_{12}^{-1}ds^2(\mathbb{E}^3) + ds^2(\mathbb{E}^4)] , \tag{II.18}$$

and the 4-form field strength remains the same. This solution provides a new embedding of the $D = 5$ black hole with non-zero entropy. By calculating the horizon area A_H, we obtain the following expression for the Bekenstein-Hawking entropy (we have introduced the five-dimensional Newton constant)

$$S_{BH} = \frac{A_H}{4G_N^5} = \frac{A_3}{4G_N^5}\sqrt{\alpha_1\alpha_2\alpha_3} = \frac{A_3}{4G_N^5}\sqrt{(|Q_1P_2| + |Q_2P_2|)\,|Q_3|}, \tag{II.19}$$

where Q_3 is the electric charge associated with the Kałuża-Klein(KK) modes along the y_1 direction in (II.18). In the quantum mechanical D-brane theory the charges Q_i and P_i become quantised (this fact follows from the quantisation of the angle ζ and parameters α_i), e.g., $|Q_3| = c_3q_3$ where c_3 is the quantum of charge and q_3 a positive integer. We then expect the entropy to have the following integer valued form

$$S_{BH} = 2\pi\sqrt{(q_1p_2 + q_2p_2)\,q_3} . \tag{II.20}$$

Compactifying this solution, as in the previous subsection, we obtain the D-brane $(2 \perp 4|4 \perp 2)$ bound state (II.16) with an extra momentum along the Y^1 direction. Applying T-duality along Y^2 we obtain the $(1_D \perp 5_D|3 \perp 3)$ bound state[3] described by the following D-branes

$$\begin{aligned} 3 &: Y^1, Y^4, Y^5 , \\ &\quad F_{45} = -\cot\zeta , \\ 5 &: Y^1, Y^2, Y^3, Y^4, Y^5 , \\ &\quad F_{45} = \tan\zeta , \end{aligned} \tag{II.21}$$

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[3]We use the subscripts D (Dirichlet), S (solitonic) and F (fundamental) whenever there is ambiguity, i.e. in the Type IIB theory and for $p = 1, 5$.

carrying momentum along the Y^1 direction. This bound state interpolates between the D-brane configuration used by Callan and Maldacena [24] in their black hole entropy counting, i.e. the $1_D \perp 5_D$ configuration, and the $3 \perp 3$ configuration. It would be extremely interesting to obtain the entropy formula (II.19) by using the D-brane counting associated with the configuration described by (II.21).

Finally, let us note that by adding a KK monopole [12] along one of the directions of $\mathbb{E}^4$ in (II.18) we have an embedding of the four-dimensional black hole with non-zero entropy.

Another $D = 4$ embedding can be obtained by performing the electromagnetic duality described at the beginning of this section on the $2 \perp 2 \perp 5 \perp 5$ configuration [9].[4] This can be done by identifying the three $\mathbf{u}$ directions in (II.1) with the three common directions to both five-branes. The resulting entropy is[5]

$$S_{BH} = \frac{A_2}{4G_N^4} \sqrt{\alpha_1 \alpha_2 \alpha_3 \alpha_4} = \frac{A_2}{4G_N^4} \sqrt{|Q_1 Q_2 P_3 P_4| + |P_1 P_2 Q_3 Q_4|} \; . \tag{II.22}$$

where

$$Q_i = \cos\zeta \; \alpha_i \; , \quad P_i = -\sin\zeta \; \alpha_i \; , \quad i = (1,2) \; ,$$

$$P_i = \cos\zeta \; \alpha_i \; , \quad Q_i = \sin\zeta \; \alpha_i \; , \quad i = (3,4) \; . \tag{II.23}$$

Charge quantisation should give the entropy an integer valued form as in (II.20).

III. BRANES AT ANGLES AND THEIR DUALS

In this section, we start with known solutions, i.e., two orthogonally intersecting p-branes, and use a solution generating technique to obtain new solutions that are interpreted as configurations of D-p-branes (as well as other p-branes) intersecting at an angle (different from $\pi/2$ or zero). The existence of these former configurations has been deduced in [28] by using D-brane quantum mechanics, and the corresponding supergravity solutions correspond to those in [29]. Here, we generate such solutions of Type IIA supergravity by reducing the $2 \perp 2$ and $2 \perp 5$ configuration of the M-theory along a specific boost direction and further duality transformations. We will find new T-duals of the branes intersecting at an angle and comment on the corresponding D-brane theory.

A. D-p-branes intersecting at an angle

Let us start with the $2 \perp 2$ M-brane configuration [7,8]

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[4]The same solution can be obtained by applying T-duality at an angle on the $3 \perp 3 \perp 3 \perp 3$ Type IIB solution.

[5]Note that this solution is $SL(2,\mathbb{Z})$ transformed of the basic four charge configurations with two electric and two magnetic charges [30].

$$ds_{11}^2 = (\tilde{H}_1\tilde{H}_2)^{\frac{1}{3}} \left[-(\tilde{H}_1\tilde{H}_2)^{-1}dt^2 + \tilde{H}_1^{-1}ds^2(\mathbb{E}_1^2) \right.$$

$$\left. +\tilde{H}_2^{-1}ds^2(\mathbb{E}_2^2) + du^2 + ds^2(\mathbb{E}^5) \right] \ , \tag{III.1}$$

$$\mathcal{F} = -d(\tilde{H}_1^{-1} - 1) \wedge \epsilon(\mathbb{M}_1^3) - d(\tilde{H}_2^{-1} - 1) \wedge \epsilon(\mathbb{M}_2^3) \ ,$$

where we conveniently parameterise $\tilde{H}_i = \sin^2\zeta + \cos^2\zeta\ H_i$ and H_i are harmonic functions on the Euclidean space $\mathbb{E}^5$. Next, perform the following coordinate (boost) transformation [20]:

$$\begin{aligned} t &\to \ \cos^{-1}\zeta\,(t + \sin\zeta\ u) \ , \\ u &\to \ \cos^{-1}\zeta\,(u + \sin\zeta\ t) \ , \end{aligned} \tag{III.2}$$

where we conveniently relate the boost parameter β to the parameter ζ (in the modified harmonic functions $\tilde{H}_i$), as $\cosh\beta = \cos^{-1}\zeta$. Reducing along the u direction yields a *static* configuration of Type IIA theory:

$$ds_{10}^2 = H_{12}^{\frac{1}{2}} \left[-H_{12}^{-1}dt^2 + \tilde{H}_1^{-1}ds^2(\mathbb{E}_1^2) + \tilde{H}_2^{-1}ds^2(\mathbb{E}_2^2) + ds^2(\mathbb{E}^5) \right] \ ,$$

$$e^{2\phi} = \left(\tilde{H}_1\tilde{H}_2 \right)^{-1} H_{12}^{\frac{3}{2}} \ ,$$

$$\mathcal{F}_4 = -\cos^{-1}\zeta \left[d(\tilde{H}_1^{-1} - 1) \wedge \epsilon(\mathbb{M}_1^3) + d(\tilde{H}_2^{-1} - 1) \wedge \epsilon(\mathbb{M}_2^3) \right] \ , \tag{III.3}$$

$$\mathcal{H} = -\tan\zeta \left[d(\tilde{H}_1^{-1} - 1) \wedge \epsilon(\mathbb{E}_1^2) + d(\tilde{H}_2^{-1} - 1) \wedge \epsilon(\mathbb{E}_2^2) \right] \ ,$$

$$\mathcal{F}_2 = -\sin\zeta\ d\left(H_{12}^{-1} - 1 \right) \wedge dt \ ,$$

where H_{12} is defined by $\tilde{H}_1\tilde{H}_2 = \sin^2\zeta + \cos^2\zeta\ H_{12}$. This solution interpolates between the 1/4 supersymmetric $2 \perp 2$ solution (for $\sin\zeta = 0$) and the 1/2 supersymmetric D-0-brane solution (for $\cos\zeta = 0$). In our notation it is expressed as $(2 \perp 2|0)$.

The ADM mass density can be expressed in terms of the D-2-brane charges Q_1 and Q_2, and the D-0-brane charge Q_0 as[6]

$$\frac{M}{3A_4} = \sqrt{(|Q_1| + |Q_2|)^2 + Q_0^2} \ , \tag{III.4}$$

where

$$Q_i = \cos\zeta\ \alpha_i \ , \quad Q_0 = \sin\zeta\ (\alpha_1 + \alpha_2) \ , \quad i = (1, 2). \tag{III.5}$$

---

[6]We remark that, in this case, it is not necessary to take the modulus of Q_1 and Q_2. However, if we had started with 2-branes with opposite charge in (III.1) that would be the case.

More generally, by applying T-duality [41] along the overall transverse space of the solution (III.3) we obtain the D-brane non-threshold bound states

$$\left(p \perp p | p - 2 \right) , \tag{III.6}$$

for $2 \leq p \leq 7$. We will comment on the D-brane theory describing this configuration later.

In order to obtain the supergravity solution interpreted as D-2-branes intersecting at an angle we perform a T-duality transformation along one direction of each D-2-brane in the configuration described by (III.3), say along y_1 and y_3. The resulting solution is

$$
\begin{aligned}
ds_{10}^2 = \ & H_{12}^{-\frac{1}{2}} \left[-dt^2 + \tilde{H}_1 dy_1^2 + 2 \sin \zeta \cos \zeta \ (1 - H_1) dy_1 dy_2 \right. \\
& + \tilde{H}_1^{-1} \left(H_{12} + \sin^2 \zeta \cos^2 \zeta \ (1 - H_1)^2 \right) dy_2^2 \\
& + \tilde{H}_2 dy_3^2 + 2 \sin \zeta \cos \zeta \ (1 - H_2) dy_3 dy_4 \\
& \left. + \tilde{H}_2^{-1} \left(H_{12} + \sin^2 \zeta \cos^2 \zeta \ (1 - H_2)^2 \right) dy_4^2 \right] + H_{12}^{\frac{1}{2}} ds^2(\mathbb{E}^5) ,
\end{aligned}
\tag{III.7}
$$

$$e^{2\phi} = H_{12}^{\frac{1}{2}} ,$$

$$
\begin{aligned}
\mathcal{A}_3 = \ & - \cos \zeta \ H_{12}^{-1} dt \wedge \left[\tilde{H}_2 (H_1 - 1) dy_2 \wedge dy_3 + \tilde{H}_1 (H_2 - 1) dy_1 \wedge dy_4 \right] \\
& + \sin \zeta \ \left(H_{12}^{-1} - 1 \right) dt \wedge dy_1 \wedge dy_3 \\
& + \sin \zeta \cos^2 \zeta \ H_{12}^{-1} (H_1 - 1)(H_2 - 1) dt \wedge dy_2 \wedge dy_4 .
\end{aligned}
$$

Note that the off-diagonal components of this metric arise, after the T-duality transformations, from the NS-NS 2-form gauge potential $\mathcal{B}$ in (III.3). We claim that this solution should be interpreted as the intersection at an angle of two sets of parallel D-2-branes. In fact, taking $H_2 = 1$ (and therefore $\tilde{H}_2 = 1$ and $H_{12} = H_1$) this solution is interpreted as D-2-branes along y_3 and making an angle $\frac{\pi}{2} - \zeta$ with the y_1 direction in the $y_1 y_2$ plane. Similarly, taking $H_1 = 1$ we have D-2-branes along y_1 and making an angle $\frac{\pi}{2} - \zeta$ with the $y_3 y_4$ plane.

This configuration constitutes a special case of the type of solutions presented in [29]. To be more precise, after a rotation in the $y_1 y_2$ plane we were able to obtain the solution (20) of [29].[7] If the harmonic functions are centered at the same point, the metric in (III.7) can be diagonalized and this solution is seen to be the standard $2 \perp 2$ D-brane solution

---

[7] Note that a different kind of configuration interpreted as D-branes intersecting at an angle bounded to a fundamental string was obtained in [42]. There, in general, the metric cannot be diagonalized by a constant $SO(2)$ rotation.

[43]. The configuration at angles can thus be brought into an orthogonal one by a constant $SO(2)$ rotation (which is part of the U-duality symmetry group).

It is interesting that we have been able to deduce the existence of this configuration by starting with the $2 \perp 2$ M-brane configuration and by following a duality orbit. In the spirit of [19], the existence of branes at angles may be seen as a consequence of diffeomorphism invariance of the underlying theories.

Importantly, for $\zeta \neq \pi/2$ the configuration preserves $1/4$ of supersymmetry; for the special case $\sin \zeta = 0$ it is the $2 \perp 2$ D-brane configuration. On the other hand, for $\zeta = \pi/2$ (i.e. $\cos \zeta = 0$) the angle between the intersecting D-2-branes is zero and the number of preserved supersymmetries is increased. In this limit the configuration is the $1/2$ supersymmetric D-2-brane configuration.[8]

The ADM mass density is given by

$$M = \frac{1}{3A_4} \left(\alpha_1 + \alpha_2 \right) \ , \tag{III.8}$$

which is the sum of the constituent D-2-brane masses. Therefore this is a D-brane bound state at threshold. Defining the electric charges [29]

$$Q_{ij} = \frac{1}{3A_4 V_{ij}} \int_{\Sigma_{ij}} \star \mathcal{F} \ , \tag{III.9}$$

where the integration is over the asymptotic 4-sphere and the compactified $y_i y_j$ space with volume V_{ij}, we have

$$Q_{24} = \sin \zeta \left(\alpha_1 + \alpha_2 \right) \ , \quad Q_{14} = -\cos \zeta \ \alpha_1 \ , \quad Q_{23} = -\cos \zeta \ \alpha_2 \ . \tag{III.10}$$

In terms of these charges the ADM mass can be written as

$$\frac{M}{3A_4} = \sqrt{\left(|Q_{14}| + |Q_{23}| \right)^2 + Q_{24}^2} \ . \tag{III.11}$$

Of course applying T-duality along the overall transverse directions in (III.7) we obtain solutions interpreted as D-p-branes intersecting at an angle for $2 \leq p \leq 7$.

Let us now comment on the D-brane configuration $(2 \perp 2|0)$ described by the solution (III.3). It is related to the D-2-branes intersection at an angle (Eq. (III.7)) by T-duality transformations along the y_1 and y_3 directions. The latter configuration can be represented as

$$\begin{aligned} 2_1 &: \ Y^3, Y^1 = \tan \zeta \ Y^2 \ , \\ 2_2 &: \ Y^1, Y^3 = \tan \zeta \ Y^4 \ . \end{aligned} \tag{III.12}$$

Applying T-duality along the Y^1 and Y^3 directions we obtain the D-brane theory describing the $(2 \perp 2|0)$ bound state

---

[8] From the D-brane world-volume approach the enlargement of supersymmetry as the angle between D-branes approaches zero and the role of the world-volume gauge fields in this process, was studied in [40].

$$2_1 : Y^1, Y^2 ,$$
$$F_{12} = -\tan \zeta ,$$
$$2_2 : Y^3, Y^4 ,$$
$$F_{34} = -\tan \zeta . \tag{III.13}$$

Note that if one of the world-volume 2-form gauge fields, associated with a given set of D-2-branes, were not turned on the corresponding configuration would not be supersymmetric.

Next, we rewrite the D-2-brane configuration (III.12) in the following way

$$2_1 : Y^3, Y^2 = \cot \zeta \; Y^1 ,$$
$$2_2 : Y^1, Y^4 = \cot \zeta \; Y^3 , \tag{III.14}$$

and apply T-duality along the Y^2 and Y^4 directions. The resulting configuration interpolates between the 1/4 supersymmetric $2 \perp 2$ D-brane configuration and the 1/2 supersymmetric D-4-brane configuration, as can be seen by studying the corresponding supergravity solution (the off-diagonal terms in the metric in (III.7) will be traded for the NS-NS 2-form gauge potential). This $(2 \perp 2|4)$ D-brane bound state is then described by

$$4_1 : Y^1, Y^2, Y^3, Y^4 ,$$
$$F_{12} = \cot \zeta ,$$
$$4_2 : Y^1, Y^2, Y^3, Y^4 ,$$
$$F_{34} = \cot \zeta . \tag{III.15}$$

We remark that this is not the same bound state found by Lifschytz [44] (without the extra D-0-brane) whose classical 1/2 supersymmetric counterpart solution has been found in [18]. Such a configuration couples to a D-0-brane due to the coupling term $tr \left(\int F \wedge F \wedge \mathcal{A} \right)$ in the D-4-brane world-volume action, which in turn requires that the F_{21} and F_{43} components of the world-volume 2-form gauge field strength are associated with the same set of D-4-branes as opposed to our configuration.

Of course the above set of solutions generalise to the

$$\left(p \perp p | (p+2) \right) \tag{III.16}$$

non-threshold D-brane bound states for $2 \leq p \leq 6$.

B. Other p-branes intersecting at an angle

We now consider the reduction along a boost of the $2 \perp 5$ M-brane solution (II.12) that we rewrite in the following way

$$ds_{11}^2 = \tilde{H}_1^{\frac{1}{3}} \tilde{H}_2^{\frac{2}{3}} \left[\left(\tilde{H}_1 \tilde{H}_2 \right)^{-1} ds^2(\mathbb{M}^2) + \tilde{H}_1^{-1} dy_2^2 \right.$$

$$\left. + \tilde{H}_2^{-1} ds^2(\mathbb{E}^4) + du^2 + ds^2(\mathbb{E}^3) \right] , \tag{III.17}$$

$$\mathcal{F} = -d \left(\tilde{H}_1^{-1} - 1 \right) \wedge \epsilon(\mathbb{M}^2) \wedge dy_2 - \star d\tilde{H}_2 \wedge dy_2 \wedge du .$$

Performing the coordinate transformation (III.2) and reducing along the u direction we obtain the Type IIA solution

$$ds_{10}^2 = \left(\tilde{H}_2 H_{12}\right)^{\frac{1}{2}} \left[-H_{12}^{-1}dt^2 + \left(\tilde{H}_1\tilde{H}_2\right)^{-1} dy_1^2\right.$$

$$\left. +\tilde{H}_1^{-1}dy_2^2 + \tilde{H}_2^{-1}ds^2(\mathbb{E}^4) + ds^2(\mathbb{E}^3)\right] ,$$

$$e^{2\phi} = \tilde{H}_1^{-1}\tilde{H}_2^{-\frac{1}{2}}H_{12}^{\frac{3}{2}} , \quad \mathcal{F}_2 = -\sin\zeta\, d\left(H_{12}^{-1} - 1\right) \wedge dt ,$$

$$\mathcal{F}_4 = -\cos^{-1}\zeta\, d\left(\tilde{H}_1^{-1} - 1\right) \wedge dt \wedge dy_1 \wedge dy_2$$

$$-\sin\zeta\cos\zeta\, \star dH_2 \wedge dy_2 \wedge dt ,$$

$$\mathcal{H} = -\cos\zeta\, \star dH_2 \wedge dy_2 - \tan\zeta\, d\left(\tilde{H}_1^{-1} - 1\right) \wedge dy_1 \wedge dy_2 ,$$

$$\text{(III.18)}$$

where H_{12} is defined as in (III.3). This solution interpolates between the $2 \perp 5$ solution (for $\sin\zeta = 0$) and the D-0-brane solution (for $\cos\zeta = 0$), i.e. it is the $(2 \perp 5|0)$ bound state.

Next, apply T-duality along the y_1 direction. The resulting Type IIB solution is

$$ds_{10}^2 = \tilde{H}_2^{\frac{1}{2}}H_{12}^{-\frac{1}{2}} \left[-dt^2 + \tilde{H}_1 dy_1^2 + 2\sin\zeta\cos\zeta\,(1 - H_1)dy_1 dy_2\right.$$

$$\left. +\tilde{H}_1^{-1}\left(H_{12} + \sin^2\zeta\cos^2\zeta\,(1 - H_1)^2\right) dy_2^2\right]$$

$$+\tilde{H}_2^{\frac{1}{2}}H_{12}^{\frac{1}{2}}\left[\tilde{H}_2^{-1}ds^2(\mathbb{E}^4) + ds^2(\mathbb{E}^3)\right] ,$$

$$e^{2\phi} = H_{12} , \quad \mathcal{B} = -\cos\zeta\,\omega \wedge dy_2 ,$$

$$\mathcal{A}_4 = -\sin\zeta\cos\zeta\, H_{12}^{-1}\,\omega \wedge dt \wedge dy_1 \wedge dy_2$$

$$-\tan\zeta\,\left(\tilde{H}_2^{-1} - 1\right) \wedge \epsilon(\mathbb{E}^4) ,$$

$$\mathcal{A}_2 = \sin\zeta\,\left(H_{12}^{-1} - 1\right) dt \wedge dy_1 - \cos\zeta\,\tilde{H}_2 H_{12}^{-1}\,(H_1 - 1)\, dt \wedge dy_2 ,$$

$$\text{(III.19)}$$

where $d\omega = \star dH_2$ and $\star$ is the dual with respect to the Euclidean metric in $\mathbb{E}^3$. Note that the second term in $\mathcal{A}_4$ arises by imposing the self-duality condition $\mathcal{F}_5 = \star\mathcal{F}_5$ by hand, where now $\star$ is the dual with respect to the ten-dimensional metric.

This solution should be interpreted as a D-string intersecting the $(5_S|1_D)$ bound state at an angle. In fact, taking $H_2 = 1$ (and therefore $\tilde{H}_2 = 1$ and $H_{12} = H_1$) this solution corresponds to a D-string in the $y_1 y_2$ plane making an angle $\frac{\pi}{2} - \zeta$ with the y_1 direction. Similarly, taking $H_1 = 1$ we have the bound state of a D-string lying within a solitonic 5-brane, i.e. the $(5_S|1_D)$ bound state. If $\sin\zeta = 0$ we have the $1_D \perp 5_S$ solution and if $\cos\zeta = 0$ the D-string solution. Note that an S-duality transformation will generate a

configuration interpreted as a fundamental string intersecting a $(5_D|1_F)$ bound state at an angle.

As in the case of the solution (III.7), the metric in (III.19) can be diagonalized if the harmonic functions are centered at the same point. However, in this case this solution is different to any standard orthogonally intersecting p-brane solution. Since the constituent p-branes are charged with respect to different fields, they are still distinguished by the supergravity solution as opposed to the case (III.7).

Since our main goal is to find bound states of D-branes, we proceed by performing a T-duality transformation along one of the directions in $\mathbb{E}^4$ belonging to the solitonic 5-brane. As expected the corresponding Type IIA theory solution is interpreted as a D-2-brane intersecting a $(5|2)$ bound state at an angle. Next, we lift this solution to $D = 11$ and reduce it along a direction internal to the 5-brane (of M-theory). The resulting solution is interpreted as a D-2-brane intersecting a $(4|2)$ bound state at an angle. The background fields are given by

$$
\begin{aligned}
ds_{10}^2 =\ & H_{12}^{-\frac{1}{2}} \left[-dt^2 + dy_5^2 + \tilde{H}_1 dy_1^2 + 2\sin\zeta\cos\zeta\ (1 - H_1) dy_1 dy_2 \right. \\[2mm]
& \left. + \tilde{H}_1^{-1} \left(H_{12} + \sin^2\zeta\cos^2\zeta\ (1 - H_1)^2 \right) dy_2^2 \right] \\[2mm]
& + H_{12}^{\frac{1}{2}} \left[\tilde{H}_2^{-1} ds^2(\mathbb{E}^2) + ds^2(\mathbb{E}^4) \right]\ ,
\end{aligned}
$$

$$
e^{2\phi} = H_{12}^{\frac{1}{2}} \tilde{H}_2^{-1}\ , \qquad \mathcal{B} = \tan\zeta\ \left(\tilde{H}_2^{-1} - 1 \right) \wedge \epsilon(\mathbb{E}^2)\ ,
$$

$$
\begin{aligned}
\mathcal{A}_3 =\ & \cos\zeta\ \omega \wedge dy_2 + \sin\zeta\ \left(H_{12}^{-1} - 1 \right) dt \wedge dy_1 \wedge dy_5 \\[2mm]
& - \cos\zeta\ \tilde{H}_2 H_{12}^{-1} \left(H_1 - 1 \right) dt \wedge dy_2 \wedge dy_5\ ,
\end{aligned}
$$

(III.20)

where $d\omega = \star dH_2$ and now H_i are harmonic functions on $\mathbb{E}^4$.

The claimed interpretation for this solution can be seen by taking the appropriate limits: for $H_2 = 1$ we have a D-2-brane along the y_5 direction and making an angle $\frac{\pi}{2} - \zeta$ with the y_1 direction in the $y_1 y_2$ plane; for $H_1 = 1$ we have the bound state of a D-2-brane in the $y_1 y_5$ plane lying within a D-4-brane in the $y_1 y_5$ and $\mathbb{E}^2$ planes. This configuration interpolates between the $2 \perp 4$ and D-2-brane configurations and can be represented as $(2 \perp 4|2)$. As in (III.9) we can define the electric charges

$$
Q_{ijk} = \frac{1}{2 A_3 V_{ijk}} \int_{\Sigma_{ijk}} \star \mathcal{F}\ , \tag{III.21}
$$

where the integration is over the asymptotic 3-sphere and the compactified $y_i y_j y_k$ space with volume V_{ijk}. With this definitions the non-zero charges are

$$
Q_{234} = \sin\zeta\ (\alpha_1 + \alpha_2)\ , \quad Q_{134} = -\cos\zeta\ \alpha_1\ . \tag{III.22}
$$

The magnetic charge carried by the D-4-brane is

$$P_2 = -\cos\zeta \; \alpha_2 \; . \tag{III.23}$$

The ADM mass density is then seen to be

$$M = \frac{1}{2A_3}(\alpha_1 + \alpha_2) = \frac{1}{2A_3}\sqrt{\left(|P_2| + |Q_{134}|\right)^2 + Q_{234}^2} \; , \tag{III.24}$$

which is the sum of the masses of the intersecting branes, i.e. the D-2-brane and the $(4|2)$ D-brane bound state.

We now comment on the D-brane theory describing the solution (III.20). This can be done by realizing that this D-brane configuration is in fact T-dual to a configuration of D-3-branes intersecting at an angle. To be more precise, applying T-duality along Y^5 and Y^3 on the configuration (III.12) we obtain the desired result

$$\begin{aligned} 2 &: \; Y^5, Y^1 = \tan\zeta \; Y^2 \; , \\ 4 &: \; Y^5, Y^1, Y^3, Y^4 \; , \\ &\quad F_{34} = -\tan\zeta \; , \end{aligned} \tag{III.25}$$

which confirms our interpretation for this D-brane bound state. More generally, applying T-duality along Y^5 or along the overall transverse space directions we can have a D-p-brane intersecting a $(p+2|p)$ bound state at an angle for $1 \leq p \leq 6$.

A new solution interpreted as a D-4-brane intersecting a $(2|4)$ bound state at an angle can be obtained by performing T-duality along two orthogonal directions of $\mathbb{E}^2$ in (III.20). In fact, such solution is just the electro-magnetic dual of the solution (III.20) and could be obtained by using the methods described in section two. The resulting configuration interpolates between the $4 \perp 2$ and the D-4-brane configurations, and can be represented as $(4 \perp 2|4)$. The corresponding D-brane theory is obtained by performing T-duality transformations along the Y^3 and Y^4 directions in (III.25)

$$\begin{aligned} 4_1 &: \; Y^3, Y^4, Y^5, Y^1 = \tan\zeta \; Y^2 \; , \\ 4_2 &: \; Y^3, Y^4, Y^5, Y^1 \; , \\ &\quad F_{34} = \cot\zeta \; . \end{aligned} \tag{III.26}$$

More generally, applying T-duality along Y^5 or along the overall transverse space directions we can have a D-p-brane intersecting a $(p-2|p)$ bound state at an angle, for $3 \leq p \leq 8$.

Finally, a T-duality transformation along Y^2 and Y^5 will bring the configuration (III.25) to

$$\begin{aligned} 2 &: \; Y^1, Y^2 \; , \\ &\quad F_{12} = \cot\zeta \; , \\ 4 &: \; Y^1, Y^2, Y^3, Y^4 \; , \\ &\quad F_{34} = -\tan\zeta \; , \end{aligned} \tag{III.27}$$

which can be generalised to

$$\left((p-2) \perp (p+2)|p \right) \tag{III.28}$$

D-brane configuration for $2 \leq p \leq 6$.

C. More black holes with non-zero entropy

As in section two, we use the above set of configurations at angles in order to obtain new embeddings of the supersymmetric black holes with non-zero entropy. This can be done by realizing that both D-brane configurations (III.25) and (III.26) admit a momentum along the Y^5 direction. In terms of the corresponding supergravity solutions this fact translates into to the possibility of adding a plane wave along the common direction to all constituent branes. The resulting solutions provide new embeddings of the $D = 5$ black hole with non-zero entropy. Consider, as an example, the supergravity solution (III.20) with a plane wave along the y_5 direction. By calculating the area of the horizon (in the Einstein frame) we obtain the following expression for the Bekenstein-Hawking entropy

$$S_{BH} = \frac{A_3}{4G_N^5} \sqrt{\cos^2 \zeta \ \alpha_1 \alpha_2 \alpha_3} = \frac{A_3}{4G_N^5} \sqrt{|Q_{134} P_2 Q_3|} \ , \tag{III.29}$$

where the charges Q_{134} and P_2 are defined in (III.22) and (III.23), and Q_3 is the electric charge associated with the KK modes in the y_5 direction. We note that the entropy is generally non-vanishing (it vanishes for the special case $\cos \zeta = 0$) and is in a non-trivial way related to the D-2-brane direction in the $Y^1 Y^2$ plane, as well as to the D-4-brane world-volume gauge field (see (III.25)).

$D = 4$ black holes with finite entropy can be obtained by adding a KK monopole which will yield the same structure of the entropy:

$$S_{BH} = \frac{A_2}{4G_N^4} \sqrt{\cos^2 \zeta \ \alpha_1 \alpha_2 \alpha_3 \alpha_4} = \frac{A_2}{4G_N^4} \sqrt{|Q_{134} P_2 Q_3 P_4|} \ , \tag{III.30}$$

where P_4 is the magnetic charge associated with the KK monopole. The quantisation constraints on ζ and α_i are expected to ensure that the entropies (III.29) and (III.30) take the form of the type: $S_{BH} = 2\pi \sqrt{q_{123} p_2 q_3}$ and $S_{BH} = 2\pi \sqrt{q_{123} p_2 q_3 p_4}$, respectively, where $(q_{123}, \ q_3, \ p_2, \ p_4)$ take integer values.

In order to make contact with the D-brane configuration used in the black hole entropy counting in [24] we perform a T-duality transformation along Y^2 on the D-brane configuration (III.25). The corresponding supergravity solution interpolates between the $1_D \perp 5_D$ and the D-3-brane solutions. For the sake of comparison with (II.21) we arrange this new D-brane configuration in the following way

$$\begin{aligned} 3 : \ & Y^1, Y^4, Y^5 \ , \\ & F_{45} = \cot \zeta \ , \\ 5 : \ & Y^1, Y^2, Y^3, Y^4, Y^5 \ , \\ & F_{23} = -\tan \zeta \ , \end{aligned} \tag{III.31}$$

where the D-branes carry momentum along the Y^1 direction. It is interesting that this D-brane configuration and the D-brane configuration (II.21) differ only by their world-volume 2-form gauge field strengths. However, their interpretation as non-threshold intersections of D-branes is quite different. The Bekenstein-Hawking entropy (III.29) is now written as

$$S_{BH} = \frac{A_3}{4G_N^5} \sqrt{\cos^2 \zeta \ \alpha_1 \alpha_2 \alpha_3} = \frac{A_3}{4G_N^5} \sqrt{|Q_1 P_2 Q_3|} \ , \tag{III.32}$$

where Q_1 is the charge associated with the D-string that is bounded to the D-3-brane, P_2 is the D-5-brane charge and Q_3 the charge associated with the KK modes along the Y^1 direction.

We remark that this entropy does depend on the angle parameter ζ; its value *decreases* as this parameter increases, and it approaches zero as this parameter approaches $\pi/2$. The microscopic picture in terms of the D-brane configuration is described by non-zero values of the world-volume gauge fields in (III.31), which *decrease* the contribution to the microscopic entropy and render the entropy zero as $\cos\zeta \to 0$ (In this limit the world-volume 2-form gauge field strengths in (III.31) approach zero and infinity, respectively.). In conclusion, the new parameter ζ associated with the D-brane world-volume gauge fields, probes the microscopic structure of these more general black holes.

IV. CONCLUSIONS

In this paper we have constructed new supergravity p-brane solutions of superstring theory and M-theory. This was done by using the $SL(2,\mathbb{R})$ electro-magnetic duality of the $D = 8$, $N = 2$ supergravity or the dimensional reduction along a boost on configurations of two orthogonally intersecting M-branes. In particular, we have shown that there is a multitude of new BPS D-brane bound states with non-vanishing worldvolume gauge fields. Specifically, we have concentrated on configurations of two p-branes intersecting at an angle different from $\pi/2$.

The main motivation for our work was to provide new D-brane configurations that are relevant for black hole physics. In fact, the microscopic origin of the black hole entropy in terms of D-brane configurations, when the corresponding world-volume gauge fields form a condensate [as in the cases (II.21) and (III.31)], opens an interesting avenue in the study of black hole physics in string theory. In particular, black holes which are obtained upon compactification of D-brane configurations intersecting at an angle, have the entropy (e.g., (III.29) and (III.30)) which explicitly depends on this angle. The same (quantised) parameter in turn specifies the world-volume gauge field condensates (III.31).

Another interesting problem is to study the relationship between the stringy description of these new D-brane configurations and their world-volume gauge theories. The appearance of many new features is expected, as it is already the case for the special cases considered in [37].

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